Vibration and Coulomb Friction

Overview

This explanation has been assembled to demonstrate the effects of internal and/or external vibration (oscillation) as applied to any generic structure resting on a plane. For example, let us consider "an audio component" sitting directly on a shelf or equipment rack and apply a resonant force or oscillation as in **Figure 1**.

> The proof below (in most simple of terms) will show that a singular vibration, when applied to a component, will dissipate over time and not in an instantaneous manner due to the effects of Coulomb friction alone.

Note that any device directly isolating a structure from the plane on which it lies will dramatically increase the effects of Coulomb friction. This will affect the slope of the resulting function (as will be derived), thus extending the total time period for an applied oscillation to completely decay.

The mathematical example expanded upon within this document will work in the "real world" with any material resting upon any surface, but we will keep audio in mind in order to maintain this verification within our realm of interest.

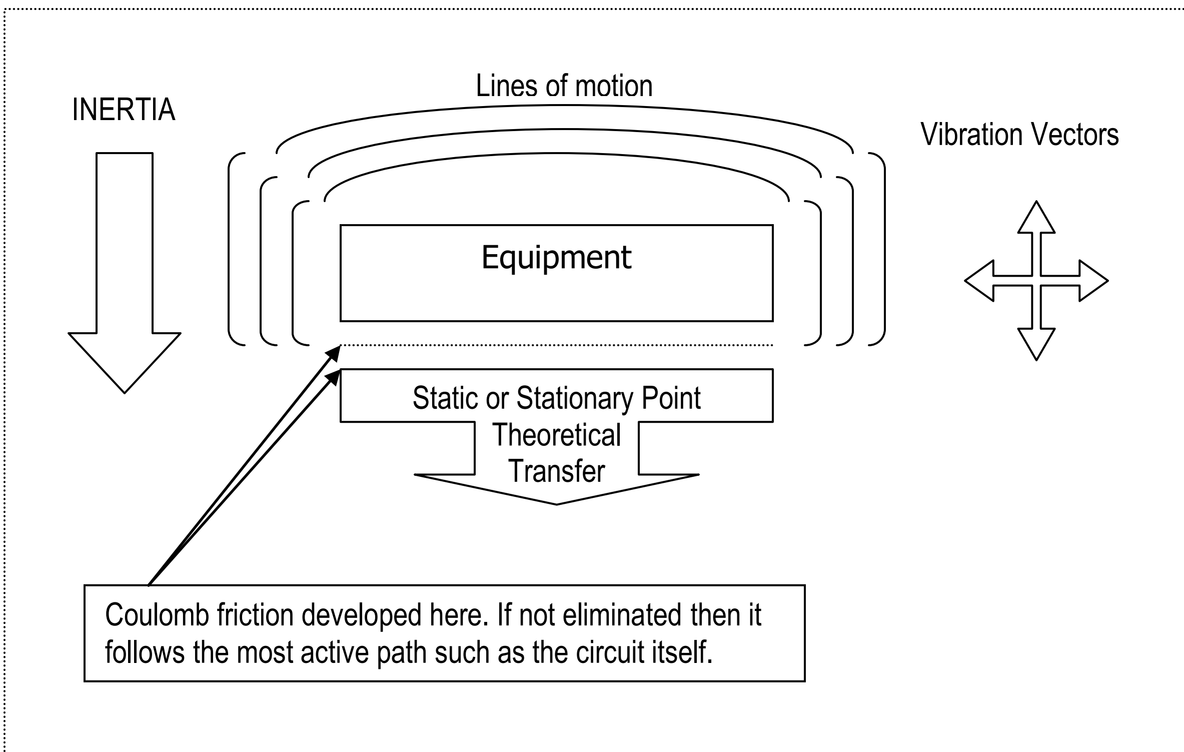

Figure 1: High-level model of Coulomb friction

Preface

Start by defining a model:

The block below represents any generic structure (piece of equipment – audio/video component, etc.) sitting on a plane or "surface". Actually, let us analyze at a molecular level; the block below can simply represent any point or particle of the component as it mates with the adjoining surface.

To demonstrate the general movement of the block's material as caused by vibration, we will employ a spring attached to the block. This will emulate an oscillation source (airborne resonance, for example) as well as offer us a visual reference. The motion of the spring is equivalent to the characteristics of an applied resonant force, except for frequency bandwidth, of course, which is a factor that does not mathematically affect the overall conclusion of the proof below.

Again, for the sake of simplicity, let us also consider this example in a one-degree-of-freedom system according to **Figure 2**. Keep in mind as you subject this system to multiple degrees of freedom and multiple instances of complex frequencies (i.e. the "real world"), the complexity of the mechanical degradation and effects of Coulomb friction are amplified exponentially.

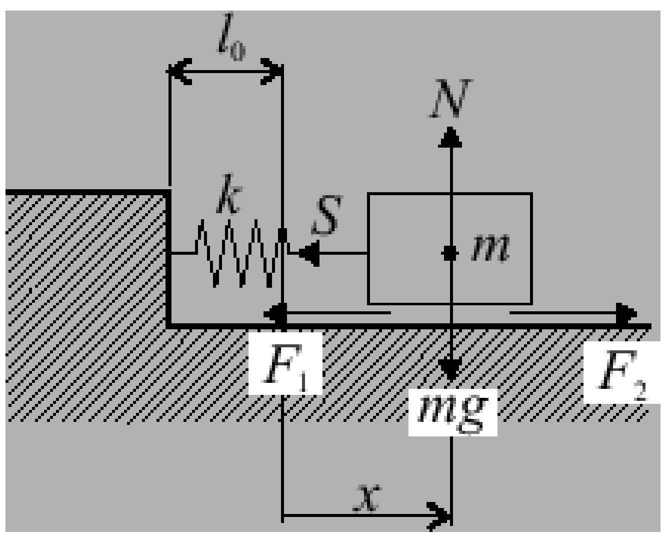

Figure 2: Vibration with Coulomb Friction

Proof

For a given mass ***m***, spring stiffness ***k***, coefficient of friction ***f***, and initial condition for **t=0**, $\mathbf{x(0)=X_0}$ and $\mathbf{x^{\bullet}(0)=0}$, the aim is to determine the motion of the mass as a function of time.

The Coulomb friction (where the friction force is proportional to the normal reaction, **N**) is considered.

Let us denote the motion to the right (in the positive direction of the ***x*** axis) as **1**, and the opposite one as **2**. The equation of motion in the direction **1** is

$$\mathbf{m\ddot{x} = -kx - F_1} ,$$

while in the direction **2** we have

$$\mathbf{m\ddot{x} = -kx - F_2} .$$

The friction forces are

$$\mathbf{F_1 = F_2 = N}\boldsymbol{f}\mathbf{ = mg}\boldsymbol{f} .$$

Introducing $\mathbf{\Omega^2 = k/m}$ and rearranging we get

$$\mathbf{\ddot{x} + \Omega^2 x = \pm g}\boldsymbol{f} .$$

The solution can be assumed in the form

$$\mathbf{x = A}\ \boldsymbol{sin}\mathbf{(\Omega t + \gamma) \pm g}\boldsymbol{f}\mathbf{/\Omega^2} .$$

The plus sign holds true for the motion in the direction **2**, the minus sign for direction **1**.

For the prescribed initial conditions, given above, the motion in the direction **2** will come up first.

Then the particle displacement and velocity allow expressing the unknown constants

$$\mathbf{x = A}\ \boldsymbol{sin}\mathbf{(\Omega t + \gamma) + g}\boldsymbol{f}\mathbf{/\Omega^2} \quad \rightarrow \quad \mathbf{X_0 = A}\ \boldsymbol{sin}\ \mathbf{\gamma + g}\boldsymbol{f}\mathbf{/\Omega^2}$$

$$\mathbf{x^{\bullet} = A\Omega}\ \boldsymbol{cos}\ \mathbf{(\Omega t + \gamma)} \quad \rightarrow \quad \mathbf{0 = A\Omega}\ \boldsymbol{cos}\ \mathbf{\gamma} .$$

So,

$$\cos \gamma = \mathbf{0} \quad \rightarrow \quad \mathbf{\gamma = \pi / 2}$$

$$\mathbf{x_0} = A + \mathbf{g}\boldsymbol{f}\mathbf{/\Omega^2} \quad \rightarrow \quad \mathbf{A = x_0 - g}\boldsymbol{f}\mathbf{/\Omega^2} .$$

Introducing

$$\mathbf{p = g}\boldsymbol{f}\,/\,\mathbf{\Omega^2}$$

For the first half-period we can write

$$\mathbf{x = (x_0 - p)}\ \boldsymbol{cos}\ \mathbf{(\Omega t) + p}\ \ .$$

At the end of the first half-period, $\mathbf{t_1 = T/2 = \pi / 2}$, the particle stops and begins to move to the right (direction **1**) with new initial conditions, i.e.

$$\mathbf{x(t_1) = (x_0 - p)}\ \boldsymbol{cos}\ \mathbf{(\Omega\pi / \Omega) + p = -(x_0 - 2p)}\ \ ,$$

$$\mathbf{x^{\bullet}(t_1) = 0}\ \ .$$

Now the displacement of the particle for motion in the direction **1** is

$$\mathbf{x = A}\ \boldsymbol{sin}\ \mathbf{(\Omega t + \gamma) - p}\ \ .$$

Substituting the new initial conditions (a), (b) in the equation of motion (c) and into its time derivative

$$\mathbf{x^{\bullet} = A\Omega}\ \boldsymbol{cos}\ \mathbf{(\Omega t + \gamma)} \qquad \text{(a)}$$

We get new integration constants valid for the second half-period, namely

$$\cos \gamma = \mathbf{0} \quad \rightarrow \quad \gamma = \mathbf{\pi / 2}\ \ , \qquad \text{(b)}$$

$$\mathbf{-x_0 + 2p = A - p} \quad \rightarrow \quad \mathbf{A = -(x_0 - 3p)}\ \ . \qquad \text{(c)}$$

So the displacement of the particle for motion in the direction **1** is

$$\mathbf{x = -(x_0 - 3p)}\ \boldsymbol{cos}\ \mathbf{(\Omega t) - p}\ \ .$$

Note: Notice that time $\boldsymbol{t}$ is measured from $\mathbf{t_1}$ from now on. Assume that for another half-period the particle displacement is described by

$$\mathbf{x = -(x_0 - 4p)}\ \boldsymbol{cos}\ \mathbf{(\Omega t + \gamma) + p}\ \ .$$

Expand this explanation over multiple periods and the result set would yield a graph as shown below:

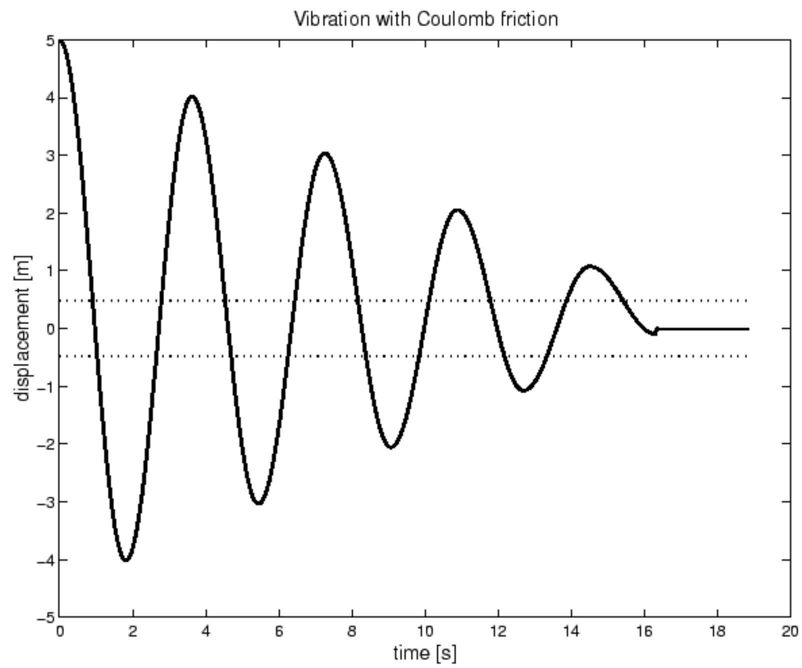

Figure 3: Displacement of a particle under the influence of Coulomb friction

One should notice that the amplitudes of vibration (**Figure 3**) decrease linearly; the motion stops sometime after the amplitude of displacement becomes less than **±p** .

Conclusion and Application

Essentially, Coulomb friction (or forced-damped vibration) is not considered to be acknowledged here unless it is a repeated resonance or of an oscillatory pattern. The above text is taken with a simplistic approach considering a singular part of a signal.

In the terms of music or other complex signals such as keyboard synthesizers, strings, drums, etc., many of the stated parameters are required to be addressed to each part of the signal simultaneously. This creates very complex harmonics which can carry out to several or multiple levels of harmonics, noting that there will be a reduction in amplitude over time.

Simply stated, if the vibration is not directed away from the source, it will modulate the source causing the original signal to appear distorted, thus defining the requirement for continuous dissipation of vibration.

In a practical application scenario, you have multiple electronic components and loudspeakers connected through a series of conduits (defined as instruments), all of which interact with each other producing sound and picture. We describe this union as a system. The system generates and conducts various forms of resonance, including mechanical, electrical, and airborne.

Refer to "The Proving Grounds" for more details on these basic principles online at http://www.starsound.biz/proving.html.

This form of resonant energy is indirectly reintroduced to the system and will cause interference with the signal pathway (being that of the whole) at one or more points in the audio/visual reproduction process. If not addressed, these resonance patterns will propagate forming inefficiencies thus limiting the product's function and affecting the overall sound and/or visual quality.

Employing isolation techniques, one merely protects one component from interfering with another that it is in direct contact with. Isolation increases the effects of Coulomb friction by building resistance between the mating surfaces. With regards to airborne resonance, isolation principles serve much like the dielectric material in a capacitance device, essentially turning the component into a giant Resonance Capacitor.

This is not the opinion of our company, Star Sound Technologies, LLC but rather that of the average graduate - level physics textbook.

Minimizing the resistance (as caused by Coulomb friction) via a mechanical grounding process is the only logical way to compensate for the effects of Mechanical, Electrical and Airborne resonance within any given system.

Again, if detrimental vibration is not directed away from a component, loudspeaker or cable conduit, it will modulate and re-modulate the original signal causing degradation of said signal. This defines the requirement for continuous dissipation (not isolation) of the unwanted vibration.

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